# Finite Element Synthesis

Neri Oxman
*Mediated Matter Group, MIT Media Lab*

## ABSTRACT

Finite element applications have customarily been used as a means for the analysis of constant-property prototypes prior to their physical fabrication. To a lesser extent they have been made instrumental for purposes of design generation and fabrication of variable material-property prototypes. The work explores the potential of applying finite element methods in the early stages of the design process and suggests a Finite Element Synthesis (FES) approach to the design of physical prototypes fabricated with variable material properties. The approach seeks to unify between analysis protocols and computational routines for design generation. With the aim of achieving micro structural material property variation across the surface area and volume of a fabricated component, the FES software environment is implemented in a design exploration for building skins modeled after various natural tissues that demonstrate the variation of physical properties as a function of their performance criteria. Two classes of explorations are suggested. The paper introduces the FES design approach, illustrates its virtual methodological set up, and demonstrates the approach through design explorations recently exhibited at the Museum of Science (Boston) and the Museum of Modern Art (NY). Future work into material-based analytic routines for variable-property fabrication is suggested and its implications on the various fields of design are reviewed.

## 1 INTRODUCTION

Virtual design generation for physical rapid prototyping, particularly for additive fabrication platforms, typically assumes a given homogeneous material from which the prototype is fabricated (Kruth, Leu et al. 1998; Hague, Campbell et al. 2003). Correspondingly, most additive and subtractive digital fabrication technologies assume the use of materials characterized by consistently uniform properties across their volume or surface area (Zhang, Xu et al. 2002). Inspired by formation processes in the biological world, where properly variation corresponds with environmental stimuli, this research promotes the integration of the finite-element method in a generative design context, supporting the controlled variation of micro-structural material properties as part of form-generation and digital fabrication processes. Material variations are computed as functions of structural and environmental performance criteria through the development of a material-based object-oriented finite-element software environment able to compute and assign graded physical properties as continuous property gradients of a functional component.

### 1.1 *Background*

The basic iterative algorithm for finite element optimization is based on reducing material concentration where it is not required for purposes of structural or mechanical performance, as defined by a given objective function (Zienkiewicz and Morice 1971; Johnson 1987; Hughes and Hughes 2000; Zienkiewicz and Taylor 2000; Zienkiewicz, Taylor et al. 2005). A simulated solid block of any given material will undergo formation changes during optimization resulting in structural efficiency across various product-, or building scale (Figure 1). In this iterative process, stress distribution is iteratively calculated and elements with minimum stress values are removed. Such routine is implemented to cater for a specific objective function (i.e. structural optimization) while assuming a relatively homogeneous material distribution (Zienkiewicz and Morice 1971; Zienkiewicz and Taylor 2000; Zienkiewicz, Taylor et al. 2005). The optimization algorithm may be combined with generative routines such as Genetic Algorithms (GA) to assist with fitness evaluation by implementing binary functions ("retain, or remove, material") at the scale of the element (Zwieniecki, Boyce et al. 2004).

Simplified, the general iterative algorithm for finite element structural optimization operates as follows: (1) Begin with a solid block of material and specified load conditions; (2) Run finite element analysis to calculate stress distribution; (3) Check for any elements at less than maximum allowable stress; (4) If such do not exist, then terminate, otherwise; (5) Remove some of the lowest-stressed elements; (6) Go to 2.

The effect, basically, is to remove redundant material until the minimum amount necessary to perform the structural task is left - at which point, all remaining material should be fully stressed; the algorithm then terminates. Such typical algorithm assumes materials of homogeneous properties (Figure 2). The only possible element conditions, then, are "full" of material or “empty”.

**Figure 1**. Examples of various possible implementations of FEM in product, and building scale. Analysis protocols are traditionally devoid of generative-design and fabrication capabilities.

**Figure 2**. The Mercedes-Benz Bionic Car Project. Image illustrating the Soft-Kill Optimization (SKO) method implemented on a solid block of material to form-find the automobile optimal shape. The fabricated material is characterized by homogenous properties. Source: http://www.carbodydesign.com

However, the designer may decide to vary the material properties of a functional element (e.g. variable-density metal foam), such that it can continuously vary from 0 (empty, no material) to 1 (full of the strongest available material), thus generating controlled heterogeneous material organizations both in the virtual and physical domains. Such design possibilities are at present considerably limited (Oxman 2010).

### 1.2 *Problem Definition*

Applications of the finite element method (FEM) are customarily used for the analysis of prototypes *prior* to their physical fabrication. The finite element method is a numerical technique for analyzing a computer model of a given homogeneous material that is stressed or analyzed for specific results (Norrie and De Vries 1973; Szabo and Babuška 1991; Moaveni 1999; Zienkiewicz, Taylor et al. 2005; Moaveni 2007). To a lesser extent has this method been made instrumental for purposes of design generation or fabrication, particularly when the design involves heterogeneous material properties (Oxman 2011). Central to this condition is the partitioning between design analysis, generation and fabrication, limiting the possibility of fast iterations between virtual finite element environments to digital fabrication platforms.

Traditional design processes are instead streamlined, and generally devoid of platforms supporting property-variation based analysis and fabrication (Oxman 2007; Oxman 2009).

## 2 AIMS & OBJECTIVES

### 2.1 *Aims: FEM to FAB*

This research explores the potential of [1] applying the finite element method in the very early stages of design generation and fabrication and, [2] incorporating heterogeneous material properties into FEM and FAB environments. Coupled with generative protocols, this work suggests a hybrid, *synthesis-analysis* approach to rapid fabrication platforms of heterogeneous material properties. The approach seeks to unify between analysis protocols and computational routines for rapid-fabrication, specifically additive fabrication platforms.

### 2.2 *Objective [1]: Generative Synthesis*

The objective is to apply the logic and computation of finite element approaches to the problem of design generation by considering the synthesis of form as composed of finite-element units, each analyzed according to spatial and material constraints.

### 2.3 *Objective [2]: Variable-Property Finite Element Fabrication*

Implementing the ability to synthesize and control material distribution as part of digital form-generation; this research investigates design forms that become possible when we can continuously vary material properties. This becomes particularly significant in considering not only structural optimi-

zation, but also insulating properties, transparency, and other architecturally relevant properties of materials, which can be optimized against *multiple* performance criteria (Oxman 2010).

We propose a general approach to the problem of computational form-generation of shapes with continuously varied material properties satisfying prescribed material conditions on a finite collection of material features and global constraints.

## 3 METHODOLOGY

### 3.1 *Distribution and Property-driven Finite Element Synthesis*

The challenge of inverting an analytical routine to a synthetic one requires the redefinition of the analytical unit and mesh components as *synthetic* cellular entities which can be further connected, combined, "grown" or generated to form a surface, or a volume unit.

We propose two cases for finite element synthesis within a fabrication context: [1] Distribution-driven Finite Element Synthesis: this case corresponds with objective 1, resulting in the coupling of generative design routines and FEA using additive fabrication technologies. Finite elements are spatially distributed corresponding with external constraints. [2] Property-driven Finite Element Synthesis. This case corresponds with objective 2, implementing the first objective within a variable-property material context, using additive fabrication technologies. The two cases are conceptualized in Figure 3 below.

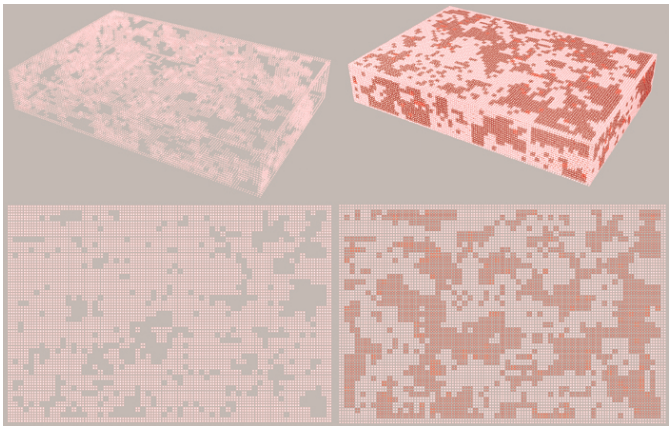

**Figure 3**. Left: distribution-driven finite element synthesis; Right: Material-property driven finite element synthesis.

### 3.2 *Object-oriented Finite Element Analysis*

We implement and further develop the Object Oriented Finite Element Analysis (OOF) environment as the basis for these experiments. OOF was developed by Prof. Craig Carter at MIT's Department of Material Science and Engineering in collaboration with the National Institute of Standards and Technology (NIST), for analyzing the effects of microstructure on material properties (Cannillo and Carter 2000; Langer, Fuller Jr et al. 2001). It serves to predict material behavior under a range of objective functions defined by the user. For any given 2-D image of material specimens, one can analyze its physical behavior based on a hypothetical assignment of physical properties to geometrical attributes (Carter, Langer et al. 1998; Carter 2010).

The computation is performed using an image-based finite element application. Physical properties are imposed onto the image after which a computational mesh is created which includes the image-property information. The computation produces various data sets including stress and strain data, heat flow, stored energy, and deformation due to applied loads and temperature differences. The results are spatially analyzed and converted to a constructible data structure using *Mathematica* (Cannillo and Carter 2000; Langer, Fuller Jr et al. 2001).

The following research utilizes OOF's ability to integrate a finite element approach with material inputs. Further development of the software to implement 3D data for fabrication was carried out as demonstrated in the design explorations (Figure 4).

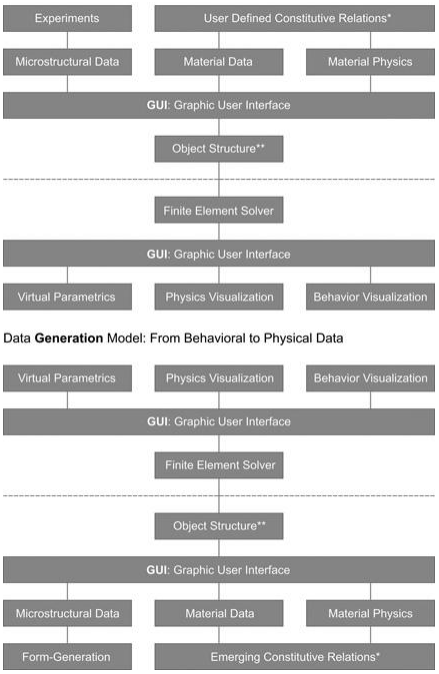


**Figure 4**. Methodological set-up implementing OOF in 3-D modeling, analysis and fabrication context.

The input files include micrographs and simulations using all available micro-structural data with no mean-field approximations. Constitutive relations translating stresses into strains using Young's modulus are defined by the user. OOF converts an image, or a micrograph, of a heterogeneous, multi-component material into a finite element mesh with constitutive properties specified by the user.

### 3.3 *Variable-Density Fabrication*

With the aim of achieving micro-, and macro-structural material property variation across the surface and volume of a fabricated component, the software environment is implemented in a design exploration for building skins modeled after various natural tissues that demonstrate the variation of physical properties as a function of their desired performance criteria. Two classes of explorations are presented that demonstrate distribution-driven and property-driven finite element synthesis and fabrication.

## 4 DESIGN EXPERIEMNTS

### 4.1 *Distribution-driven Finite Element Synthesis & Fabrication*

Imagine the case in which the size of a mesh-free particle, applied for the purpose of form-generation informed by light performance, precisely matches the size of an imaginable powder molecule, or – more realistically speaking – a material aggregate providing for the substance of the 3D printing process. Such is the design motivation behind *Raycounting* – the form of which is mediated by environmental and structural constraints.

The element unit can be thought of as an intermediary representation linking the digital form to its physical manifestation, particularly when rapid fabrication processes are considered. In this respect, the "element" provides for a lower limit material definition establishing the degree of granularity required to manifest the 3-D details of the design. From here, it is relatively easy to imagine the implications of using 'finite-synthesis elements' as the units used for calibrating voxels and 3-D printing powder. The designer is generating 3-D form using the precise units applied to describe its physical manifestation.

*Raycounting* is a prototype for a product skin designed as a variable-translucency surface. The doubly-curved 3D model is generated by integrating a 3D generative modeling environment (Rhinoceros 4) with a finite-element application to determine surface thickness and material distribution as a function of a desired array of light effects.

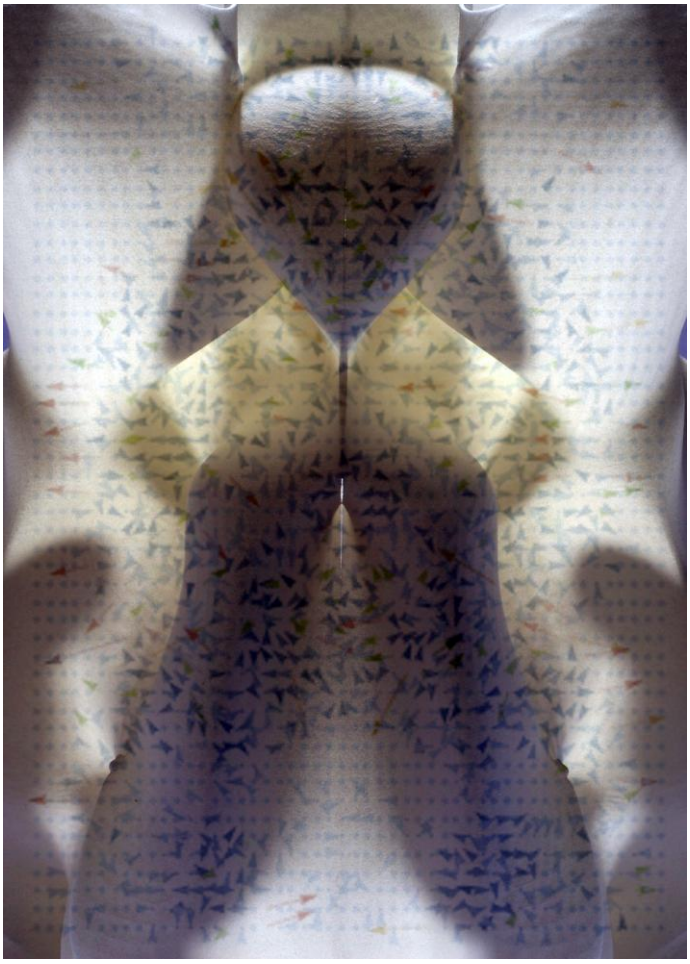

**Figure 5**. *Raycounting*, Museum of Modern Art, 2008. 3-D printed prototype embedded with color-coded light-performance data defining material distribution and surface thickness.

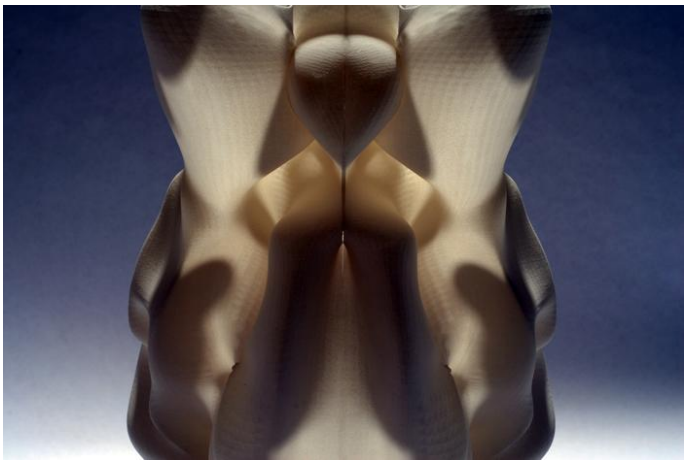

**Figure 6**. *Raycounting*, Museum of Modern Art, 2008. 3-D printed prototype demonstrating variable surface thickness informed by the Finite Element environment.

### 4.2 *Property-driven Finite Element Analysis & Fabrication*

Mesh discretization processes allow the designer to subdivide a continuous mathematical domain into a set of discrete sub-domains referred to as elements and represented as singular geometrical entities. Lattices and triangulations are common rationalization discretization techniques, where quadrant and trian-

gulated elements may respectively wrap the surface area or volume of the object. Such structural meshes are often implemented by engineers in order to simulate structural loads, analyze their distribution and predict any potential displacements that may arise.

*Fatemaps* is a prototype demonstrating the integration of variable-property material data with modeling and fabrication routines. This process allows the designer to import any physical tissue data sampled from the physical world, and embed its properties as part of a 3-D modeling and fabrication environment. The study explores natural tissues reconstruction by calibrating the size, shape and proximity of each element comprising the tissue to the size and shape of the material unit from which the form is to be fabricated (Figure 7).

**Figure 7**. *Fatemaps*, Museum of Science Boston, 2010. Right: illustration of the OOF environment further developed to include 3D data. Left: physical prototype reconstruction of a natural tissue (butterfly wing); the physical model is generated within OOF and printed using an OBJET multi-material. An integrated software environment support modeling, analysis and preparation for 3-D printing.

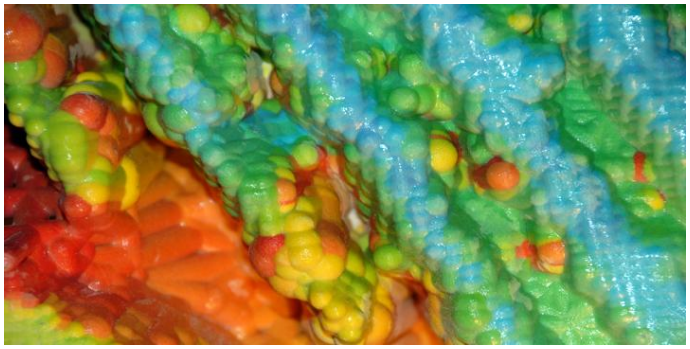

**Figure 8**. *Fatemaps*, Museum of Science Boston, 2010. Detailed view of 3-D printed tissue.

## 5 CONTRIBUTIONS

Finite element analysis methods and their practical applications are primarily applied to evaluate a given design relative to some objective function. Currently advanced applications exist that support the integration of form-generation and evaluation from a structural perspective. Such are, for instance, tools developed for automobile shape optimization routines based on the types of loads considered, their magnitudes and directions. However, despite their generative advantage, such tools have yet to incorporate *variable-material data* as part of the form-generation and fabrication processes.

The *Finite Synthesis Method (FSM)* was developed as a theoretical approach and methodology supporting the integration of modeling and analysis routines in the process of digital fabrication. It affords the designer the ability to consider analysis tools in their generative capacities while corresponding to variable-property material data. In order to further implement this approach within a design environment, the concept of *finite element synthesis* has been introduced, whereby each element as defined by the FE application may contain, in addition to its structural data, information regarding other performance criteria that are of interest to the designer. In this regard, each *material element* is regarded as a tensor element defined by indices negotiating various objective functions. This method supports the distribution of properties across the entire surface area of the design object relative to the various architectural and engineering performance criteria addressed.

Beyond the notion of performance-driven interpretations based on computational geometry methods, this work has also engaged with the notion of computational analysis as a source for strategizing material distribution. Rather than breaking down the design into a series of componentized elements aiming at straightforward and simplified assemblies, the experiments undertaken in this research demonstrate an alternative approach favoring material distribution over design strategies of composition and assembly.

## 6 FUTURE WORK

The *Finite Element Synthesis* approach assumes and facilitates the distribution of multiple material properties as a function of site-specific constraints of various types. In other words, for each material element, the size and shape of which is defined within the FE software, there exists an array of related properties specifically defined for that element. Future work will focus on inventing new fabrication platforms able to support variable material-property

fabrication in the scale of the finite element application. In this case, each ‘element may be computed as a physical entity both virtually and physically.

Currently there exist no fabrication technologies that allow for the production of objects with gradually varying structural properties at the scale of product design and architectural construction. While this work has demonstrated that such platforms are beneficial given the potential advantages in terms material and mechanical efficiencies, there are many technical challenges yet to overcome. It is clear however, that an integrated approach to form-generation where fabrication processes play an active role in the form-generation and analysis process, promotes more efficient products and building parts to be fabricated allowing the designer to include evaluative functions early in the design process.

## 7 ACKNOWLEDGEMENTS

The author wishes to thank Professor Craig Carter, from the Department of Material Science and Engineering at MIT for his collaboration in the design projects, his insightful comments, and for providing the initial methodological set up for this work through the OOF software, which has been made available online. In addition, the author wishes to thank Professor Gibson, and Professor Ortiz from the Materials Science and Engineering Department, as well as Professor Woodie C. Flowers from the Mechanical Engineering Department, for their constant insightful input and support. The design explorations were supported by Paola Anonelli, chief curator at the Museum of Modern Art in NY. The models are now part of the permanent collection at MoMA and the Museum of Science, Boston.

## 8 PREFERENCES

Cannillo, V. and W. Carter (2000). "Computation and simulation of reliability parameters and their variations in heterogeneous materials." Acta materialia **48**(13): 3593-3605.

Carter, C.-o. s. a. J. (2010). "OOF 2: The Manual." http://www.ctcms.nist.gov/~langer/oof2man/Chapter-Overview.html.

Carter, W., S. Langer, et al. (1998). "The OOF manual: version 1.0." National Institute of Standards and Technology, Gaithersburg, MD, NISTIR **6256**.

Hague, R., I. Campbell, et al. (2003). "Implications on design of rapid manufacturing." Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science **217**(1): 25.

Hughes, T. J. R. and Hughes (2000). The finite element method: linear static and dynamic finite element analysis, Dover Publications.

Johnson, C. (1987). Numerical solution of partial differential equations by the finite element method, Cambridge university press Cambridge.

Kruth, J. P., M. Leu, et al. (1998). "Progress in additive manufacturing and rapid prototyping." CIRP Annals-Manufacturing Technology **47**(2): 525-540.

Langer, S. A., E. R. Fuller Jr, et al. (2001). "OOF: an image-based finite-element analysis of material microstructures." Computing in Science & Engineering **3**(3): 15-23.

Moaveni, S. (1999). Finite element analysis, Prentice Hall.

Moaveni, S. (2007). Finite element analysis theory and application with ANSYS, Prentice-Hall, Inc. Upper Saddle River, NJ, USA.

Norrie, D. H. and G. De Vries (1973). The finite element method: fundamentals and applications, Academic Press.

Oxman, N. (2007). "Get Real Towards Performance-Driven Computational Geometry." International Journal of Architectural Computing **5**(4): 663-684.

Oxman, N. (2009). Material-based design computation: Tiling behavior.

Oxman, N. (2010). "Material-based design computation."

Oxman, N. (2010). "Structuring Materiality: Design Fabrication of Heterogeneous Materials." Architectural Design **80**(4): 78-85.

Oxman, N. (2011). "Variable property rapid prototyping." Virtual and Physical Prototyping **6**(1): 3-31.

Szabo, B. A. and I. Babuška (1991). Finite element analysis, Wiley-Interscience.

Zhang, H., J. Xu, et al. (2002). "Fundamental study on plasma deposition manufacturing." Surface and Coatings Technology **171**(1-3): 112-118.

Zienkiewicz, O., R. Taylor, et al. (2005). The Finite Element Method–Its Basis and Fundamentals, volume 1, Butterworth-Heinemann.

Zienkiewicz, O. C. and P. Morice (1971). The finite element method in engineering science, McGraw-Hill London.

Zienkiewicz, O. C. and R. L. Taylor (2000). The finite element method: Solid mechanics, Butterworth-Heinemann.

Zwieniecki, M. A., C. K. Boyce, et al. (2004). "Functional design space of single-veined leaves: role of tissue hydraulic properties in constraining leaf size and shape." Annals of Botany **94**(4): 507.